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CONVOLUTION EQUATIONS AND MEAN VALUE THEOREMS FOR SOLUTIONS OF LINEAR ELLIPTIC EQUATIONS WITH CONSTANT COEFFICIENTS IN THE COMPLEX PLANE

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Convolution equations generated by distributions with compact supports and the corresponding mean value theorems was investigated by many authors. In particular, Volchkov described a wide class of radial distributions with compact supports such that solutions of the corresponding convolution equations in open Euclidean balls can be efficiently characterized in terms of the Bessel functions. This characterization implies different corollaries such as uniqueness theorems for solutions of the corresponding convolution equations and two-radius theorems that go back to John (1934) and Delsarte (1961), respectively.

Let s and m be nonnegative integers, s< m, and let 0<r<R. We study smooth functions f of a complex variable that are defined in the disk D(0, R) of radius R centered at zero and satisfy the convolution equation

$$
\sum_{p=s}^{m-1} \frac{\nu^{2p+2}}{(2p+2)(p-s)!p!} \partial^{p-s} \partial^p f(z) = \frac{1}{2\pi} \int_{|\zeta-z| \leq r} f(\zeta)(\zeta-z)^s d\lambda_2(\zeta), \quad (1)
$$
for all \( z \in D(0, R-r) \), where \( \partial \) and \( \bar{\partial} \) are the Cauchy formal derivatives (operators of complex differentiation) and \( \lambda_2 \) is the planar Lebesque measure.

We characterize such functions in terms of the representation of the Fourier coefficients of the function \( g(z) = \partial^{m-s}\bar{\partial}^{m}f(z) \) by series of special functions. A simple corollary of this characterization is a two-radius theorem characterizing solutions of the elliptic equation \( \partial^{m-s}\bar{\partial}^{m}f(z) = 0 \).

A remarkable feature of the convolution equation (1) is that for \( s>0 \) this equation is generated by non-radial distributions. We reduce this case to the investigation of some concrete radial distributions with compact supports and their spherical transformations that allows to apply Volchkov's results on the representation of solutions of convolution equations generated by radial distributions with compact supports.

**Література**

2. Delsarte J. Lectures on Topics in Mean Periodic Functions and the Two-Radius Theorem, Tata Institute of Fundamental Research, Bombay, 1961.

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**REMOVABLE ISOLATED SINGULARITIES FOR SOLUTIONS OF ANISOTROPIC POROUS MEDIA EQUATION**

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For the quasilinear parabolic equation in the divergent form

\[
\begin{align*}
    u_t - A(x,t,u,\nabla u) &= b(x,t,u,\nabla u), \quad (x,t) \in \Omega_T \setminus \{(x_o,0)\}, \\
    u(x,0) &= 0, \quad x \in \Omega \setminus \{x_o\},
\end{align*}
\]

(1)

(2)

where \( A(a_1,a_2,\ldots,a_n), \ b(x,t,u,\zeta) \) satisfy the following structure conditions

\[
A(x,t,u,\zeta) \geq \nu_1 \sum_{i=1}^{n} |u|^{m_i-1} |\zeta|^2,
\]

\[
|a_i(x,t,u,\zeta)| \leq \nu_1 |u|^{\frac{m_i-1}{2}} \left( \sum_{j=1}^{n} |u|^{m_j-1} |\zeta|^2 \right)^{\frac{1}{2}}, \quad i = 1, n,
\]

(3)

\[
|b(x,t,u,\zeta)| \leq \nu_2 |u|^{\frac{m-1}{2}} \left( \sum_{i=1}^{n} |u|^{m_i-1} |\zeta|^2 \right)^{\frac{1}{2}},
\]